François Patte
Université Paris Descartes
Paris

`francois.patte@mi.parisdescartes.fr`

Mathematical algorithms in Sanskrit prosody and music treatises

Mathematics in ancient civilisations is mostly seen as a discipline used for astronomy. Similarly in the Indian *śāstras*, *gaṇita* is a part of *jyotiṣa*. Although a majority of the mathematical developments stemmed from the need for a theoretical and practical tool to study the planet and astral motions, mathematics quickly became an end-in-itself, leading to its becoming a means to investigate numerous other fields, from sophisticated computations to games.

In India, to which we most likely owe the decimal place value system for numeration and operations, mathematics developed an art of computation and a science of algorithms to a large extend, probably thanks to an analytical turn of mind best examplified in Panini's grammar.

Reading prosody treatises, like Kedāra's, or *saṃgīta* treatises, like Śārṅgadeva's, we come across certain gems of this art of algorithms. In chapter six of *Vṛttaratnākara* we find a comprehensive study on how to combine long and short syllables in a metre of a fixed length. In *Saṃgītaratnākara*, chapter four, we find no less a comprehensive study on how to combine four elementary unit of time in a given measure. All these issues fall in the mathematical field of combinatorics.

For example, there are sixteen possibilities to combine long and short syllables in a four-syllable metre and there are nineteen possibilities to combine *druta*, *laghu*, *guru* and *pluta* in a musical sequence made up of six *drutas*.[1] The comprehensive enumeration of these combinations is called *prastāra* in Sanskrit treatises.

This paper intends to show that the algorithms used by the Indian *paṇḍits* to establish the *prastāras* are an ingenious construction which involves a high knowledge in mathematics: binary expansion of a number for *prastāras* in prosody, integer partitions and order structures for those in music. Moreover, these algorithms, by their simple implementation, allow many further developments such as: counting, using recursive formulæ *(saṃkhyā)*, finding the pattern of a combination by knowing its order in the *prastāra* *(naṣṭa)* or, conversely, finding the order of a given combination *(uddiṣṭa)*.

Paris, December 18, 2008

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[1] *druta* is the shortest note value; one *laghu* is two *drutas*, one guru is two *laghus* and one *pluta* is three *laghus*.